

# Fort Street High School



**2021**

ASSESSMENT TASK 4 - ONLINE EXAMINATION

## Mathematics Extension 1

Total Marks: 38

Working time: 65 minutes

**General  
Instructions**

- This is an open book task
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet will be provided
- Show relevant mathematical reasoning and/or calculations

**Specific  
Instructions:**

- You will not be able to leave your desk for the duration of the task.
- Mobile phones must be turned off and out of sight.
- Your microphone and cameras must be on, but you can turn the volume on your devices down so that any noise from other students does not disturb you.
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- You can ask for assistance through the direct chat function of Zoom/Teams or ask as your microphone is on.
- You are to manage your time and make sure you have a timer at hand to keep within the assessment time limit.
- At the end of the assessment, you will have 15 minutes to scan and submit your task on Google Classroom. During this time, if you have any difficulty with submitting your task, please communicate this with your teacher immediately.
- Please note that submission of the task is your responsibility.

**Attempt Questions 1-6**

Answer each question in a new writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

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**Question 1** (7 marks) Start a new writing booklet**Marks**

- (a) Show that the coefficient of  $x^5$  in the polynomial  $P(x) = \left(x - \frac{1}{x}\right)^{11}$  is equal to  $-165$ . **3**
- (b) (i) Express  $\sqrt{3} \cos x - \sin x$  in the form  $R \cos(x + \alpha)$ , where  $R > 0$  and  $0 < \alpha < \frac{\pi}{2}$ . **2**
- (ii) Hence solve  $\sqrt{3} \cos x - \sin x = \sqrt{2}$  for  $0 \leq x \leq 2\pi$ . **2**

**End of Question 1**

**Question 2** (8marks) Start a new writing booklet**Marks**

(a) Consider the claim:  $1 + 2 + 3 + \dots + n = \frac{1}{2} \left( n + \frac{1}{2} \right)^2$  is true for all positive integer  $n$ .

(i) Assuming the claim is true for  $n = k$ , where  $k$  is a positive integer, then prove it is true for  $n = k + 1$ .

**2**

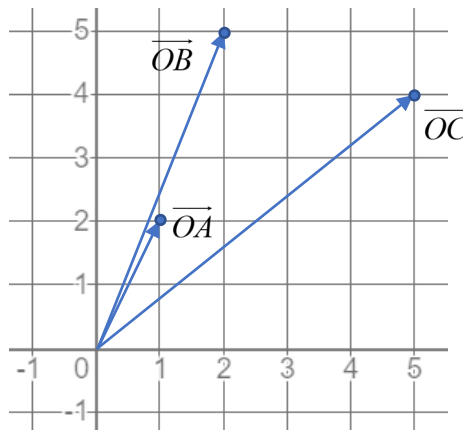
(ii) Prove the claim to be false for  $n = 1$ ?

**1**

(iii) Hence justify why the original statement is false.

**1**

(b) The coordinates of the position vectors  $\overrightarrow{OA}$ ,  $\overrightarrow{OB}$ , and  $\overrightarrow{OC}$  are  $(1, 2)$ ,  $(2, 5)$ , and  $(5, 4)$  respectively.



(i) Express vector  $\overrightarrow{AB}$  in the form  $x\mathbf{i} + y\mathbf{j}$ .

**1**

(ii) Given that  $ABCD$  is a parallelogram, use vector operations to show that the position vector  $\overrightarrow{OD} = 4\mathbf{i} + \mathbf{j}$ .

**1**

(iii) By using properties of vectors, demonstrate that  $ABCD$  is a rectangle.

**2****End of Question 2**

**Question 3** (7 marks) Start a new writing booklet

**Marks**

(a) Use the substitution  $u = x \ln x$  to show that  $\int_1^2 \frac{\ln x^2 + 2}{(x \ln x + 1)^2} dx = \frac{\ln 16}{1 + \ln 4}$  . **3**

(b) A ball rolling along a horizontal plane has position vector  $\underline{r} = x\underline{i} + y\underline{j}$ ,  $y \geq 0$

and velocity vector  $\underline{\dot{r}} = \frac{1}{y}\underline{\dot{i}} + (1 - y)\underline{\dot{j}}$  .

(i) The component of velocity in the  $\underline{j}$  direction gives the differential equation

$$\frac{dy}{dt} = 1 - y .$$

Show that the solution to this differential equation is  $\ln|1 - y| = A - t$

where  $A$  is a constant. **1**

(ii) Given that the ball is initially at the origin and that the  $y$  values are restricted to  $0 \leq y < 1$  find the velocity vector when  $t = 3$ . **3**

**End of Question 3**

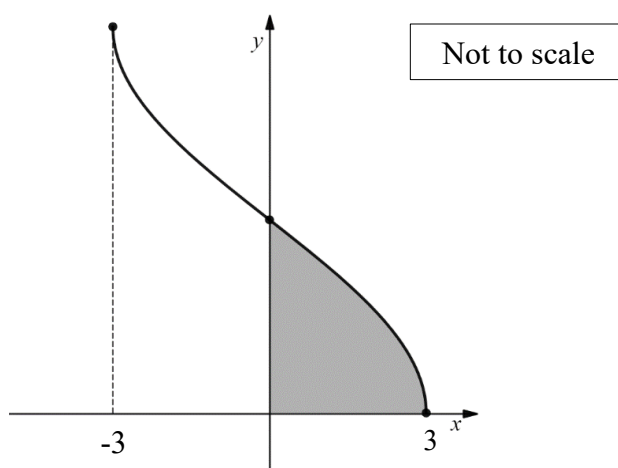
**Question 4** (5 marks) Start a new writing booklet

**Marks**

- (a) There are 21 Junior students in the library (consisting of Years 7, 8, 9, and 10). Explain why there must be at least six students from one of the year groups in the library.

**1**

- (b) The sketch shows the graph of  $f(x) = 2 \cos^{-1} \frac{x}{3}$ .



- (i) Show that the y-intercept is  $y = \pi$ .

**1**

- (ii) The shaded area is rotated about the y-axis. Find the volume of the solid obtained

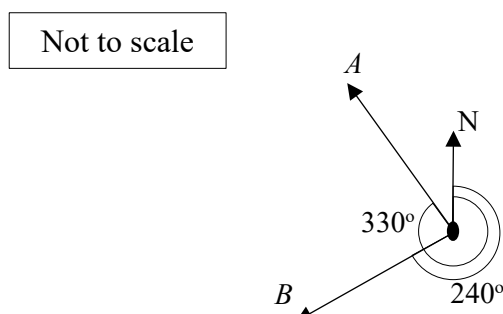
**3**

**End of Question 4**

**Question 5** (5 marks) Start a new writing booklet

**Marks**

- (a) Naveen sells his handmade bracelets at the local market. One of his bracelets comprises of 3 black, and one of each of the colours red, yellow, pink, green, and orange beads. How many variations of this bracelets can he make? **1**
- (b) During a movie scene a camera is attached to three drones by inextensible chords, completely stretched horizontally, with no vertical movement. Drone *A* is applying a force of 30N on a bearing of  $330^\circ$ , while drone *B* is applying a force of 36N on a bearing of  $240^\circ$ , as shown in the diagram.



If the camera is to be kept stationary, calculate the bearing of drone *C* and the force it needs to apply to the camera, giving your answers to 2 decimal places. **4**

**End of Question 5**

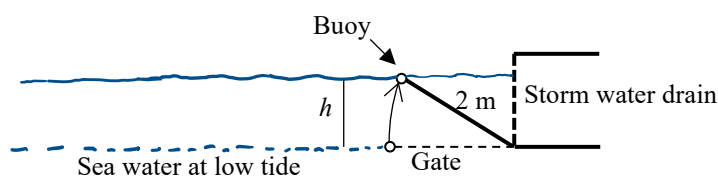
**Question 6** (6 marks) Start a new writing booklet

**Marks**

- (a) If  $P(x) = x^3 + 2x^2 + 3x + p$  and  $Q(x) = x^3 + x^2 + 9$  leave the same remainder when divided by  $x + 2$ , what is the value of  $p$ ?

**2**

- (b) A two-metre gate is attached at its one end to the base of a storm water drain on a beach while a buoy is attached to its other end. The base of the drain is aligned with the sea water at its lowest tide. As the tide rises, so does the buoy in an arc as shown in the diagram:



If the tide is rising at a rate of 0.3 cm per minutes, find the rate of change in the path of the buoy (arc length) when the tides has risen by 1.2 metres.

**4**

**End of Paper**



**2021**

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**Question 1** (7 marks) Start a new writing booklet**Marks**

- (a) Show that the coefficient of  $x^5$  in the polynomial  $P(x) = \left(x - \frac{1}{x}\right)^{11}$  is equal to  $-165$ . **3**

**Solutions:****Method 1**

$$\begin{aligned}
 T_{n+1} &= \binom{11}{n} (x)^n \left(-\frac{1}{x}\right)^{11-n} \\
 &= \binom{11}{n} (-1)^{11-n} (x)^n (x^{-1})^{11-n} \\
 &= \binom{11}{n} (-1)^{11-n} (x)^{n-(11-n)}
 \end{aligned}$$

Equating indices:

$$\begin{aligned}
 2n - 11 &= 5 \\
 2n &= 16 \\
 n &= 8
 \end{aligned}$$

Therefore:

$$\begin{aligned}
 \text{Coefficient} &= \binom{11}{8} (-1)^3 \\
 &= -165
 \end{aligned}$$

- 1** – the general term expression
- 2** – finding  $n$  from the general term **or** provided the whole term
- 3** – correct expression for the coefficient

**Markers Feedback****Method 2**

$$\begin{aligned}
 T_{n+1} &= \binom{11}{n} (x)^{11-n} \left(-\frac{1}{x}\right)^n \\
 &= \binom{11}{n} (-1)^n (x)^{11-n} (x^{-1})^n \\
 &= \binom{11}{n} (-1)^n (x)^{(11-n)-n}
 \end{aligned}$$

Equating indices:

$$\begin{aligned}
 11 - 2n &= 5 \\
 2n &= 6 \\
 n &= 3
 \end{aligned}$$

Therefore:

$$\begin{aligned}
 \text{Coefficient} &= \binom{11}{3} (-1)^3 \\
 &= -165
 \end{aligned}$$

- (b) (i) Express  $\sqrt{3} \cos x - \sin x$  in the form  $R \cos(x + \alpha)$  Where  $R > 0$  and  $0 < \alpha < \frac{\pi}{2}$ . 2
- (ii) Hence solve  $\sqrt{3} \cos x - \sin x = \sqrt{2}$  for  $0 \leq x \leq 2\pi$ . 2

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                        |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p><b>Solutions:</b></p> <p>(i) <math>\sqrt{3} \cos x - \sin x = R \cos(x + \alpha)</math></p> $R = \sqrt{3+1} = 2 \qquad \alpha = \tan^{-1} \frac{1}{\sqrt{3}} = \frac{\pi}{6}$ <p>(ii)</p> $\sqrt{3} \cos x - \sin x = \sqrt{2}$ $2 \cos\left(x + \frac{\pi}{6}\right) = \sqrt{2}$ $\cos\left(x + \frac{\pi}{6}\right) = \frac{1}{\sqrt{2}}$ $x + \frac{\pi}{6} = \frac{\pi}{4}, \frac{7\pi}{4} \quad \text{for } \frac{\pi}{6} < x + \frac{\pi}{6} \leq 2\pi + \frac{\pi}{6}$ $x = \frac{\pi}{12}, \frac{19\pi}{12}$ | <p>(i)</p> <p>1 – for either R or angle<br/>2 – for both correct</p> <p><b>Markers Feedback</b></p> <p>(ii)</p> <p>1 – for base angle<br/>2 – for complete solution</p> <p><b>Markers Feedback</b></p> |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

**End of Question 1**

(a) Consider the claim:  $1 + 2 + 3 + \dots + n = \frac{1}{2} \left( n + \frac{1}{2} \right)^2$  is true for all positive integer  $n$ .

(i) Assuming the claim is true for  $n = k$ , where  $k$  is a positive integer,  
then prove it is true for  $n = k + 1$ .

2

(ii) Prove the claim to be false for  $n = 1$ ?

1

(iii) Hence justify why the original statement is false.

1

**Solutions:**

$$1 + 2 + 3 + \dots + n = \frac{1}{2} \left( n + \frac{1}{2} \right)^2$$

(i)

Assume true for  $n = k$ , where  $k$  is a positive integer.

$$1 + 2 + 3 + \dots + k = \frac{1}{2} \left( k + \frac{1}{2} \right)^2$$

i.e.

Prove true for  $n = k + 1$

$$1 + 2 + 3 + \dots + k + (k + 1) = \frac{1}{2} \left( (k + 1) + \frac{1}{2} \right)^2$$

i.e. prove

Proof:

$$LHS = 1 + 2 + 3 + \dots + k + (k + 1)$$

$$= \frac{1}{2} \left( k + \frac{1}{2} \right)^2 + (k + 1) \quad (\text{by assumption})$$

$$= \frac{1}{2} k^2 + \frac{1}{2} k + \frac{1}{8} + k + 1$$

$$= \frac{1}{2} k^2 + \frac{3}{2} k + \frac{9}{8}$$

$$RHS = \frac{1}{2} \left( k + 1 + \frac{1}{2} \right)^2$$

$$= \frac{1}{2} \left( k + \frac{3}{2} \right)^2$$

$$= \frac{1}{2} k^2 + \frac{3}{2} k + \frac{9}{8}$$

$$\therefore LHS = RHS$$

(ii) When  $n = 1$

$$LHS = 1 \quad RHS = \frac{1}{2} \left( 1 + \frac{1}{2} \right)^2$$

$$= \frac{9}{8}$$

$$\therefore LHS \neq RHS$$

(iii) Since false for  $n = 1$ , then the statement does not hold for all positive integer  $n$ .

(i)

1 – for substantive work leading to proof

2 – for complete proof

**Markers Feedback**

(ii)

1 – for proof

**Markers Feedback**

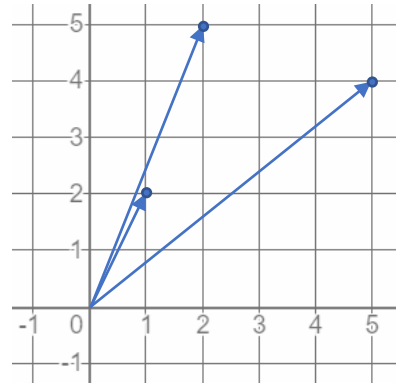
(iii)

1 – for correct reasoning

**Markers Feedback**

- (b) The coordinates of the position vectors  $\overrightarrow{OA}$ ,  $\overrightarrow{OB}$ , and  $\overrightarrow{OC}$  are (1, 2), (2, 5), and (5, 4) respectively.

- (i) Express vector  $\overrightarrow{AB}$  in the form  $x\hat{i} + y\hat{j}$ .
- (ii) Given that  $ABCD$  is a parallelogram, use vector operations to show that the position vector  $\overrightarrow{OD}$  must be  $4\hat{i} + \hat{j}$ .
- (iii) By using properties of vectors, demonstrate that  $ABCD$  is a rectangle.



1

1

2

**Solutions:**

$$\vec{a} = \hat{i} + 2\hat{j}; \quad \vec{b} = 2\hat{i} + 5\hat{j}; \quad \vec{c} = 5\hat{i} + 4\hat{j}$$

(i)  $\overrightarrow{AB} = \vec{b} - \vec{a}$

$$= (2-1)\hat{i} + (5-2)\hat{j}$$

$$= \hat{i} + 3\hat{j}$$

(ii)  $\overrightarrow{OD} = \overrightarrow{OC} - \overrightarrow{AB}$

$$= (5-1)\hat{i} + (4-3)\hat{j}$$

$$= 4\hat{i} + \hat{j}$$

(iii)  $\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$

$$= (4-1)\hat{i} + (1-2)\hat{j}$$

$$= 3\hat{i} - \hat{j}$$

$$\overrightarrow{AB} \cdot \overrightarrow{AD} = (3)(1) + (3)(-1)$$

$$= 0$$

$\therefore \overrightarrow{AB}$  is  $\perp$  to  $\overrightarrow{AD} \Rightarrow ABCD$  is a rectangle

(i)  
1 – for the answer

**Markers Feedback**

(ii) 1 – for the operation

**Markers Feedback**

(iii) 1 – for finding the necessary vectors  
2 – for dot product

**Markers Feedback**

**End of Question 2**

(a) Use the substitution  $u = x \ln x$  to show that  $\int_1^2 \frac{\ln x^2 + 2}{(x \ln x + 1)^2} dx = \frac{\ln 16}{1 + \ln 4}$  .

**3**

**Solutions:**

$$u = x \ln x$$

$$du = (1 + \ln x) dx$$

$$\text{When } x = 1 \Rightarrow u = 0$$

$$x = 2 \Rightarrow u = 2 \ln 2 (= \ln 4)$$

$$\int_1^2 \frac{\ln x^2 + 2}{(x \ln x + 1)^2} dx = \int_1^2 \frac{2(\ln x + 1)}{(x \ln x + 1)^2} dx$$

$$= \int_0^{\ln 4} \frac{2}{(u + 1)^2} du$$

$$= -2 \left[ \frac{1}{u + 1} \right]_0^{\ln 4}$$

$$= -2 \left( \frac{1}{1 + \ln 4} - 1 \right)$$

$$= -2 \left( \frac{1 - 1 - \ln 4}{1 + \ln 4} \right)$$

$$= \frac{2 \ln 4}{1 + \ln 4}$$

$$= \frac{\ln 16}{1 + \ln 4}$$

**1** – for correct substitutions and equivalent integral

**2** – for correct integration

**3** – for complete solution

**Markers Feedback**

(b) A ball rolling along a horizontal plane has position vector  $\underline{r} = x\underline{i} + y\underline{j}$ ,  $y \geq 0$

and velocity vector  $\dot{\underline{r}} = \frac{1}{y}\dot{x}\underline{i} + (1-y)\dot{y}\underline{j}$ .

(i) The component of velocity in the  $\underline{j}$  direction gives the differential equation

$$\frac{dy}{dt} = 1 - y$$

Show that the solution to this differential equation is  $\ln|1-y| = A - t$

where  $A$  is a constant.

(ii) Given that the ball is initially at the origin and that the  $y$  values are restricted to  $0 \leq y < 1$  find the velocity vector when  $t = 3$ .

1

3

(i)  $\frac{dy}{dt} = 1 - y$

$$\int dt = \int \frac{1}{1-y} dy$$

$$t = -\ln|1-y| + A$$

$$\therefore \ln|1-y| = t - A$$

(ii) When  $\left. \begin{matrix} t=0 \\ y=0 \end{matrix} \right\} \Rightarrow \ln 1 = A$

$$\therefore A = 0$$

Since  $0 \leq y < 1$  then  $\ln(1-y) = -t$

$$1-y = e^{-t}$$

$$y = 1 - e^{-t}$$

When  $t = 3$  then  $1-y = e^{-3}$  &  $\frac{1}{y} = \frac{1}{1-e^{-3}}$

Therefore, velocity vector is given by:

$$\dot{\underline{r}} = \frac{1}{1-e^{-3}}\dot{x}\underline{i} + e^{-3}\dot{y}\underline{j}$$

(i)

1 – for correct integration

**Marker Feedback**

(ii)

1 – for correct value of  $A$

2 – for correct expression for  $y$

3 – for complete solution

**Marker Feedback:**

**End of Question 3**

**Question 4** (5 marks) Start a new writing booklet

**Marks**

- (a) There are 21 Junior students in the library (consisting of Years 7, 8, 9, and 10). Explain why there must be at least six students from one of the year groups in the library.

**1**

There are 21 students and 4 groups.

$21 \div 4 > 5$ . Therefore, there must be at least one group of 6 students.

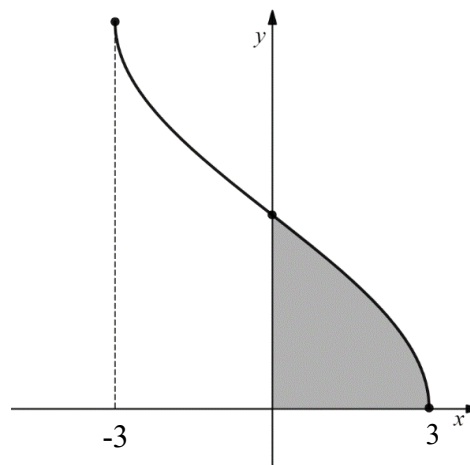
**1** – correct reasoning

**Marker Feedback**

(b) The sketch shows the graph of  $f(x) = 2 \cos^{-1} \frac{x}{3}$ .

(i) Show that the y-intercept is  $y = \pi$ .

(ii) The shaded area is rotated about the y-axis. Find the volume of the solid obtained.



1

3

(i)  $f(x) = 2 \cos^{-1} \frac{x}{3}$   
 When  $x = 0 \Rightarrow y = 2 \cos^{-1} 0$   

$$= 2 \times \frac{\pi}{2}$$
  

$$= \pi$$

(ii)  $V = \pi \int_a^b x^2 dy$  ;  $x = 3 \cos \frac{y}{2}$   

$$= \pi \int_0^{\pi} 9 \cos^2 \frac{y}{2} dy$$

Now  $\cos 2\theta = 2 \cos^2 \theta - 1$

$$\cos^2 \theta = \frac{1}{2}(\cos 2\theta + 1)$$

$$V = \frac{9\pi}{2} \int_0^{\pi} (\cos y + 1) dy$$

$$= \frac{9\pi}{2} [\sin y + y]_0^{\pi}$$

$$= \frac{9\pi}{2} [(\sin \pi + \pi) - (\sin 0 + 0)]$$

$$= \frac{9\pi^2}{2} \text{ units}^3$$

(i)  
1 – correct solution

**Markers Feedback**

(ii)  
1 – for correct setup of integral  
2 – for correct integration  
3 – for complete solution

**Markers Feedback**

**End of Question 4**

**Question 5** (5 marks) Start a new writing booklet**Marks**

- (a) Naveen sells his handmade bracelets at the local market. One of his bracelets comprises of 3 black, and one of each of the colours red, yellow, pink, green, and orange beads. How many variations of this bracelets can he make?

**1**

|                                                                                                                    |                                                     |
|--------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------|
| <b>Solutions:</b><br>$\frac{7!}{3! \times 2} = 420$ <p>Therefore, he can make 420 variations of this bracelet.</p> | <b>1 – solution</b><br><br><b>Marker's Feedback</b> |
|--------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------|

(b) During a movie scene a camera is attached to three drones by inextensible chords, completely stretched horizontally, with no vertical movement. Drone  $A$  is applying a force of 30N on a bearing of  $330^\circ$ , while drone  $B$  is applying a force of 36N on a bearing of  $240^\circ$ , as shown in the diagram. If the camera is to be kept stationary, calculate the bearing of drone  $C$  and the force it needs to apply to the camera, giving your answers to 2 decimal places.

4

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                                                                                                                                                                                                                                                 |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p> <math>\underline{A} = x_A \underline{i} + y_A \underline{j}</math><br/> <math>= -30 \cos 60^\circ \underline{i} + 30 \sin 60^\circ \underline{j}</math><br/> <math>= -15 \underline{i} + 15\sqrt{3} \underline{j}</math><br/> <math>\underline{B} = x_B \underline{i} + y_B \underline{j}</math><br/> <math>= -36 \cos 30^\circ \underline{i} + 36 \sin 30^\circ \underline{j}</math><br/> <math>= -18\sqrt{3} \underline{i} + 18 \underline{j}</math><br/> <math>\underline{C} = x_C \underline{i} + y_C \underline{j}</math> </p> <p>Since the camera is not moving:</p> $\underline{A} + \underline{B} + \underline{C} = 0$ $(-15 - 18\sqrt{3} + x_C) \underline{i} + (15\sqrt{3} - 18 + y_C) \underline{j} = 0 \underline{i} + 0 \underline{j}$ $\Rightarrow x_C = 15 + 18\sqrt{3} \quad ; \quad y_C = 18 - 15\sqrt{3}$ <p>Therefore <math> \underline{C}  = \sqrt{x_C^2 + y_C^2}</math></p> $= \sqrt{2196}$ $= 46.86149 \dots$ $\approx 46.86 \text{ N (2 d.p.)}$ <p>Therefore drone <math>C</math> must apply a force of 46.86 N on a bearing of <math>99.81^\circ</math>.</p> | <p>1 – Establishing <math>\overrightarrow{OA}</math>, <math>\overrightarrow{OB}</math> and <math>\overrightarrow{OC}</math> in component form</p> <p>2 – Applying the stabilising formula</p> <p>3 – Finding the magnitude of force or bearing</p> <p>4 – Complete solution</p> <p><b>Marker's Feedback</b></p> |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

End of Question 5

- (a) If  $P(x) = x^3 + 2x^2 + 3x + p$  and  $Q(x) = x^3 + x^2 + 9$  leave the same remainder when divided by  $x + 2$ , what is the value of  $p$ ?

**2**

$$Q(x) = x^3 + x^2 + 9$$

Using the remainder Theorem with  $Q(x)$ :

$$\begin{aligned} Q(-2) &= (-2)^3 + (-2)^2 + 9 \\ &= 5 \end{aligned}$$

Using the remainder Theorem with  $P(x)$ :

$$\begin{aligned} P(-2) &= (-2)^3 + 2(-2)^2 + 3(-2) + p \\ &= -6 + p \\ &= 5 \quad \text{From } Q(-2) \end{aligned}$$

Therefore:

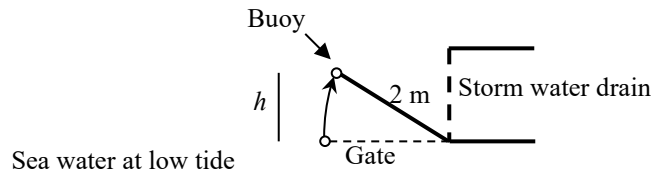
$$\begin{aligned} -6 + p &= 5 \\ p &= 11 \end{aligned}$$

**1** – for finding the remainder

**2** – for finding the value of  $p$

**Markers Feedback**

- (b) A two-metre gate is attached at its one end to the base of a storm water drain on a beach while a buoy is attached to its other end. The base of the drain is aligned with the sea water at its lowest tide. As the tide rises, so does the buoy in an arc as shown in the diagram:



If the tide is rising at a rate of 0.3 cm per minutes, find the rate of change in the path of the buoy (arc length) when the tides has risen by 1.2 metres.

4

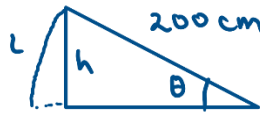
$$\frac{dh}{dt} = 0.3 \text{ cm / min ; } \quad \frac{dl}{dt} = ?$$

$$l = r\theta \quad \theta = \sin^{-1} \frac{h}{200}$$

$$= 200\theta$$

$$\frac{dl}{d\theta} = 200$$

$$\frac{d\theta}{dh} = \frac{1}{200\sqrt{1 - \left(\frac{h}{200}\right)^2}}$$



- 1 – for  $dl/d\theta$
- 2 – for  $dl/dt$  &  $d\theta/dt$
- 3 – for expression for  $dl/dt$
- 4 – for full solution

**Markers Feedback**

$$\frac{dl}{dt} = \frac{dl}{d\theta} \times \frac{d\theta}{dh} \times \frac{dh}{dt}$$

$$= 200 \times \frac{1}{200\sqrt{1 - \left(\frac{h}{200}\right)^2}} \times \frac{3}{10}$$

When  $h = 120$  cm

$$\frac{dl}{dt} = 200 \times \frac{1}{200\sqrt{1 - \left(\frac{120}{200}\right)^2}} \times \frac{3}{10}$$

$$= \frac{3}{10\sqrt{1 - \left(\frac{120}{200}\right)^2}}$$

$$= \frac{3}{8} \text{ cm/min } (= 0.375 \text{ cm/min})$$

Alternative :

$$\frac{h}{200} = \sin \theta$$

$$h = 200 \sin \theta$$

$$\frac{dh}{d\theta} = 200 \cos \theta ; \cos \theta = \frac{\sqrt{200^2 - h^2}}{200}$$

$$\frac{d\theta}{dh} = \frac{200}{200\sqrt{200^2 - h^2}}$$

$$\frac{d\theta}{dh} = \frac{1}{\sqrt{200^2 - h^2}}$$

$$\frac{dl}{dt} = \frac{dl}{d\theta} \times \frac{d\theta}{dh} \times \frac{dh}{dt}$$

$$= 200 \times \frac{1}{\sqrt{200^2 - h^2}} \times \frac{3}{10}$$

$$\frac{dl}{dt} = 200 \times \frac{1}{\sqrt{200^2 - 120^2}} \times \frac{3}{10}$$

$$= \frac{200}{160} \times \frac{3}{10}$$

$$= \frac{3}{8} \text{ cm/min } (= 0.375 \text{ cm/min})$$

**End of Paper**